Planning Autonomous Vehicles in the Absence of Speed Lanes using Lateral Potentials

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Citation: R. Kala, K. Warwick (2012) Planning autonomous vehicles in the absence of speed lanes using lateral potentials, *Proceedings of the 2012 IEEE Intelligent Vehicles Symposium*, Alcalá de Henares, Spain, pp 597-602.

Final Version Available At: http://ieeexplore.ieee.org/stamp/stamp.jsp?arnumber=06232148

***Abstract*— Chaotic traffic, prevalent in many countries, is marked by a large number of vehicles driving with different speeds without following any predefined speed lanes. Such traffic rules out using any planning algorithm for these vehicles which is based upon the maintenance of speed lanes and lane changes. The absence of speed lanes may imply more bandwidth and easier overtaking in cases where vehicles vary considerably in both their size and speed. Inspired by the performance of artificial potential fields in the planning of mobile robots, we propose here lateral potentials as measures to enable vehicles to decide about their lateral positions on the road. Each vehicle is subjected to a potential from obstacles and vehicles in front, road boundaries, obstacles and vehicles to the side and higher speed vehicles to the rear. All these potentials are lateral and only govern steering the vehicle. A speed control mechanism is also used for longitudinal control of vehicle. The proposed system is shown to perform well for obstacle avoidance, vehicle following and overtaking behaviors.**

I. Introduction

PLANNING of autonomous vehicles is an important problem which deals with deciding on the trajectory and manner in which each vehicle should be travelling. Most planning techniques (e.g. [1]-[2]) are designed to enable vehicle motion in the presence of speed lanes. A vision system is able to capture the vehicle position and orientation, which is used by an algorithm to generate control signals to enable the vehicle stay within a speed lane. A higher order planning process is used for decision making regarding changing speed lanes. In this manner the vehicles are able to show behaviors such as obstacle avoidance [3], lane changing [4], vehicle following and overtaking [5]-[6].

Planning may be broadly separated into longitudinal planning and lateral planning. Longitudinal planning deals with sticking to one's own speed lane. This involves speed control and steering control in case of curved roads. Lateral planning deals with deciding on lane changes and generating a feasible trajectory for a lane change. The planning primarily involves steering control.

Speed lanes however lead to efficient traffic motion only when vehicles are wide enough to occupy most of the speed lane in which they are travelling. When vehicles differ considerably in widths, it is possible to fit more vehicles on a road and this increases the traffic bandwidth. Having motorbikes is a clear example of allowing vehicles that can slide in between speed lanes [7] which would have otherwise occupied a speed lane of their own. When vehicles vary in their preferred speed of travel this is another common feature which leads to interesting driving behavior of overtaking. Having numerous vehicles differ in sizes and preferential driving speeds and using a road on which they defy the speed lanes constitutes chaotic traffic.

While much of the work in planning for autonomous vehicles has been done in disciplined traffic from both simulation and physical implementation points of view, the results of such plans in chaotic conditions is questionable. Indian traffic conditions in most places and most times show a clear case of chaotic traffic. It is therefore important to devise planning algorithms that work in such traffic conditions and this is the main aim of this paper. Such traffic scenarios are studied in problems of traffic prediction [8], accident analysis [9]-[10], etc. However these traffic conditions are yet to be studied from the perspective of planning of autonomous vehicles. That said, chaotic scenarios are studied in different domains which include human motion [11] and robotic motion amidst humans [12].

Little work has been done for the planning of vehicles in the absence of speed lanes. Kuwata et al. [13] used rapidly exploring random trees (RRT) for planning autonomous vehicles. The approach however did not account for the cooperation between vehicles which, as with mobile robotics, can have a large part to play in traffic dynamics. Kala and Warwick [14] employed RRT for planning of multiple vehicles using a priority based approach. However all vehicles were assumed to be autonomous with inexpensive and perfect communication. Elastic roadmaps [15] find a lot of applications for vehicle navigation and obstacle avoidance. The problems with both classes of approaches is that the map needs to be fairly well known and further cooperation is difficult to model.

Cooperative overtaking was studied by Frese and Beyerer [16], who compared mixed integer programming, tree search, elastic bands, random priorities and optimized priorities algorithms for their work. Some of the methods of the authors were on speed lane formulations, while the number of vehicles was generally lower, which questions the validity of methods in chaotic traffic. Some methods assumed good communication between all vehicles.

Artificial Potential Fields [17] have been widely used in robotics for planning the motion of a mobile robot. In this method the target is given a strong attractive potential while the obstacles have a strong repulsive potential. The gradient of potential is used to decide on the motion of the mobile robot. The ease of implementation and less computational time are the biggest advantages of this method. The method scales well to moving obstacles and other robots which may be dealt with as obstacles for decentralized robot planning. For planning multiple robots, shared potential fields [18]-[19] may be used, wherein robots may benefit from the sensor readings of the other robots and as a result they can mutually affect the movement of each other.

Though the problem of robot motion planning closely resembles the problem of planning of an autonomous vehicle in the absence of speed lanes, the potential method cannot be directly applied to vehicles. The prime reason is the presence of roads within which vehicles need to be driven. In a road scenario with moving vehicles and obstacles it would be evident to have too many zero potential points. Further cooperation is weakly modeled in potential approaches, whereas in traffic scenarios it is important for a vehicle to cooperate and allow another vehicle to overtake it. The same holds true for elastic band approaches as well.

II. Problem Description

We assume here that a map of a road segment is available in which the road is bounded by a road boundary on both sides. There can be a number of vehicles in the map at any time. The size, position and speed of nearby vehicles can be sensed by the vision system of the vehicle. There exist no speed lanes in the road and hence any vehicle can potentially drive anywhere in the road.

Let, at some time, the position of the vehicle being planned be $R(x', y', \theta')$. Here the X' axis (or longitudinal axis) is taken as the heading direction of the road and $Y'(R)$ axis (or lateral axis), at any point R, as the axis joining two boundaries normal to the X' axis. The angle θ' denoting orientation of the vehicle is measured as the angle from the $X'(R)$ axis at the point of measurement (R). The notations are shown in Fig. 1. Let the vehicle be of size l x w. Let the corners of vehicle in cyclic order be C_1, C_2, C_3, and C_4. Let the vehicle's preferential speed of driving be *vpref* which is the speed by which the vehicle travels on a straight road in the absence of any other vehicle or obstacle. Let v ($\leq$ *vpref*) be the current speed of the vehicle. Let the rotational speed of the vehicle be ω ($\leq \omega_{max}$). Here v and ω are measured in the Cartesian coordinate system which is not the system used to represent vehicle position R. The maximum acceleration that the vehicle can have is *accmax*.

The objective of the algorithm is to move the vehicle at every instant of time such that the vehicle does not collide with any static obstacle and to ensure that no two vehicles collide with each other. On top of this, vehicles may not go very close either to each other or to a static obstacle, which is a potential threat in driving. The traffic is assumed to possess large diversities in terms of the constituent vehicles. This means that vehicles vary in terms of their sizes (l x w) and preferred driving speeds (*vpref*). There is no lower limit to the allowable speed, which means traffic may have extremely slow vehicles moving in it. Hence the motion of the vehicle produced by the algorithm can only be regarded as desirable if any vehicle having a higher preferred speed is able to overtake a vehicle having lower preferred speed. Overtaking is preferred to take place on the right side (left side driving rule – UK/Japan style), but this is not a mandatory condition. On wide roads a vehicle already lying to the left of a vehicle may proceed to overtake the vehicle on the left (with some cooperation from other vehicles) rather than having to go to the other side of the road to overtake. The vehicles need not arrange themselves laterally as per their preferred speeds (typical in speed lane scenarios) which leads to overtaking mostly on the right.

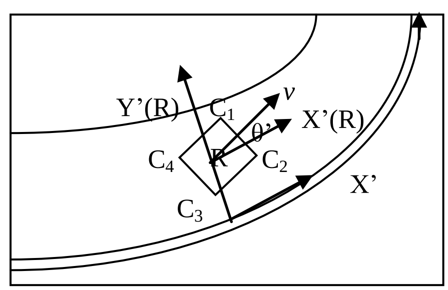

Fig. 1: Notations used for vehicle representation

III. Algorithm

The algorithm presented here is based upon the method of Artificial Potential Field which is a widely used and studied algorithm for cases of both single and multiple mobile robot planning. In this work however we prefer to model the algorithm from the perspective of the thought process of a human driver as if he/she was driving the automated vehicle. A conventional potential field design would demand using distance measures from surroundings, converting them into force vectors and moving the vehicle by the resultant force. Sonar sensors are found on a variety of robots which give the distance from obstacles directly as input and can easily be used for computation of the resultant force vector.

This methodology however does not enable us to model driving behaviors and hence a modified scheme is used. The implemented methodology enables us to generate travel plans which are more realistic, as well as to mix well in chaotic traffic comprising of both autonomous and human driven vehicles. Using this mechanism we intend to generate similar behaviors to those that are observed in countries where speed lanes are not followed. The task of planning may be easily broken down into lateral planning and longitudinal planning. While the former deals with adjusting the steering, the latter deals with adjusting the speed.

The key task of the algorithm is to decide the lateral position of the vehicle which is done using lateral potentials. The potentials may be positive, which force the vehicle to occupy a position with a larger value on the Y' axis, or negative, forcing the vehicle to go for a smaller value on the Y' axis. We use potential amalgamated from few sources to decide the resultant lateral position of vehicle.

A. *Forward Potential*

The first source of potential is from a vehicle or obstacle directly in front along the X' axis of the vehicle. Let the obstacle be at a distance of d_{fi} units away from the vehicle when measured from a point f_i on vehicle's front boundary (line C_1C_2). Let us assume that after a distance f_i longitudinally, there lies a static obstacle ($o = obs$), or another vehicle ($o = B$). The potential applied to the vehicle is given by (1).

$$p_{fi} = \begin{cases} 0 & o = B, v_B \geq vpref \quad (a) \\ \text{sgn}(B)\dfrac{vpref - v_B}{d_{fi}} & o = B, v_B < vpref \quad (b) \\ \text{sgn}(o)\dfrac{vpref}{d_{fi}} & o = obs \quad (c) \end{cases} \qquad (1)$$

Here v_b is the speed of the vehicle in front (*B*, if any). Equation 1(a) deals with the condition when the vehicle being planned (say *A*) is possibly following the vehicle ahead, vehicle *B*. There is no possibility that vehicle *A* may need to overtake vehicle B. As there is no other behavior that vehicle *A* shows because of the presence of vehicle *B*, the potential is 0.

However as per condition 1(b), overtaking is possible if vehicle *A* accelerates. Hence potential is applied in the direction of *sgn*(*B*) by vehicle *B* to vehicle *A*. *sgn*(*B*) may be +1 or -1, whose value can be determined by considering whether overtaking should take place on the left or the right. In our algorithm both sides are possible hence the value is kept 1 if *B* lies at a higher lateral position to *A* or at the same lateral position (overtaking on the right preferred), and -1 otherwise. Equation 1(c) is the same scenario where the potential is caused by a static obstacle in place of another vehicle. *sgn*(*o*) denotes the strategy to overcome the obstacle on the left (*sgn*(*o*)= 1) or the right (*sgn*(*o*)=-1).

Obstacle avoidance may be perceived as overtaking a static vehicle which accounts for the difference between 1(b) and 1(c). Unlike conventional potential approaches, prospective time to collision is used as an indicator of potential rather than distance to collision. This accounts for the commonly observed driving phenomenon wherein maneuvers are smaller on sighting a vehicle directly ahead which needs to be overtaken and larger if an obstacle is at the same distance. It may be noted that a sonar sensor may not be applicable for measuring this distance as it measures distance in heading angle of vehicle and not along the X' axis. However, knowing the positions and orientations of other vehicles and the position of vehicle *R*, this distance may be computed. The net value of potential due to front sources may be given by (2).

$$p_f = sign(max\{abs(p_{fi})\}).(max\{abs(p_{fi})\})^2, \ i \text{ lies on } C_1C_2 \qquad (2)$$

This means that the largest potential measured along any point on the front boundary is used as the front potential. This potential gives the overtaking and obstacle avoidance behavior of the vehicle. Conventional potential field modeling for a vehicle directly in front of another vehicle or obstacle would have pushed the vehicle *A* backwards instead, thereby disallowing any overtaking. On being marginally deviated in its lateral position, the lateral potential would have been too small to facilitate quick overtaking.

B. *Side Potential*

The next source of potential is an obstacle, another vehicle, or road boundary to the side of the vehicle, with distances measured along the lateral direction or the Y' axis. Let the vehicle have a distance of d_{li} (or d_{rj}) measured from a point l_i (or r_j) along Y' axis (or –Y' axis) from a point l_i (or r_j) lying at the left (or right) boundary of the vehicle that is line C_1C_4 (or line C_2C_3). The resultant potential may be given by (3).

$$p_s = p_{li} + p_{ri} = -\max\{(1/d_{li})\}^2 + \max\{(1/d_{rj})\}^2 \qquad (3)$$

i lies on C_1C_4, *j* lies on C_2C_3.

Note that speed is not mentioned in (3) unlike (2). The reason for this is that there is no concept of side speed which determines when the vehicle may collide with the sensed obstacle, road boundary or vehicle. In fact (unless the same obstacle or vehicle was sensed in (1), in which case it is governed by its dynamics), the vehicle may never collide with the obstacle, vehicle, or road boundary end, since it does not lie directly in front and the vehicle mostly moves straight longitudinally.

C. Diagonal Potential

The next source of potential is the forward diagonal distance measured at points C_1 and C_2. Consider point C_1 (or C_2) which is used to measure distance d_{flC1} (or d_{frC2}) at an angle of 45 degrees (or -45 degrees) to X' axis. This potential may be given by (4).

$$p_d = p_{fl} + p_{fr} = -(1/d_{flC1})^2 + (1/d_{frC2})^2 \quad (4)$$

The diagonal potential (p_d) acts as a forerunner to side potential (p_s). The lateral potential is recorded as a position which the vehicle would occupy in the future, if it does not make any lateral alterations. Diagonal potential enables the vehicle to make any corrections in advance.

D. Back Potential

The last source of potential is from a vehicle which may be to the rear. Let the distance be d_b in the –X' axis and a vehicle B be behind at this distance. The resultant potential is given by (5).

$$p_{bi} = \begin{cases} 0 & B = null \vee v \geq vpref_B \\ \mathrm{sgn}(B)\dfrac{vpref_B - v}{d_b} & v < vpref_B \end{cases} \quad (5)$$

In case vehicle B has a higher (than A) preferential speed ($vpref_B$) it is possible that vehicle B overtakes vehicle A. Hence vehicle A must drift towards the opposite side to which overtaking is being performed to facilitate the overtaking to take place. In our algorithm $sgn(A)$ has a value 1 if B lies at a higher lateral position to A, or at the same lateral position (overtaking on the right preferred), and -1 otherwise. The resultant potential is given by (6).

$$p_b = sign(max\{abs(p_{bi})\}).(max\{abs(p_{bi})\})^2, \ i \text{ lies on } C_3C_4 \quad (6)$$

E. Lateral Planning

There are therefore 4 sources of potential which add up to the total potential given by (7). However the different potentials are at different scales and hence cannot be directly added up.

$$p = sen_{X'}.p_f + sen_{Y'}.p_s + sen_{X'Y'}.p_d + coop.p_b \quad (7)$$

Here $sen_{X'}$ is a factor that governs the sensitivity of the vehicle from an obstacle or another vehicle directly ahead. Higher values lead to early heavy steering to avoid the obstacle or another vehicle, even though it might be way ahead. Smaller values lead to small steering early until the vehicle reaches very close to the vehicle or obstacle when left lateral corrections take place. The factor $sen_{Y'}$ governs the lateral sensitivity of the vehicle. If the factor is high the vehicle is prone to make too large steering changes for small behavioral changes. If the factor is small, the vehicle shows very slow lateral corrections and the majority of its journey is travelled in a straight line, until it reaches a state of potential collision in which case sharp steering is required.

The factor $sen_{X'Y'}$ governs sensitivity to forthcoming lateral corrections, which plays a role as a combination of the other two factors. The factor *coop* governs the degree to which the vehicle cooperates with another vehicle to the rear for potential overtaking. Small values are better for the vehicle being planned but painful for the overtaking vehicle, and vice versa.

Lateral control of the vehicle is done using the steering control which changes the orientation of the vehicle. The desired orientation of the vehicle $\theta'_{desired}$ is proportional to the lateral potential given by (8).

$$\theta'_{desired} = k.p \quad (8)$$

Here k is a constant governing conversion of potential to orientation. In practice it may not be possible to orient the vehicle to $\theta'_{desired}$ due to rotational speed restrictions ($-\omega_{max} \leq \omega \leq \omega_{max}$), in which case the maximum change possible is applied.

F. Longitudinal Planning

Longitudinally the major decision to be taken is on the speed of travel. The lateral position of the vehicle or the steering is controlled by the lateral planner, and the longitudinal planner needs to only ensure that the vehicle keeps moving at the fastest speed possible. Hence there are no longitudinal potentials used in this technique. The distance of the vehicle is measured on the X' axis.

Let the distance at any point be d_{fi}. Let (after this distance) the vehicle meet an obstacle (o=obs) or another vehicle (o=B). The corresponding maximum speed possible as a result of an obstacle being found after d_{fi} distance is given by (9).

$$v_{fi} = \begin{cases} vpref & o = B, v_B \geq vpref \quad (a) \\ \min\left(v_B + \sqrt{2(acc.agg)d_{fi}}, vpref\right) & o = B, v_B < vpref \quad (b) \\ \min\left(\sqrt{2(acc.agg)d_{fi}}, vpref\right) & o = obs \quad (c) \end{cases} \quad (9)$$

Equation 9(a) covers the case when there is no potential threat of a collision to the vehicle as no slower vehicle or static obstacle lies ahead, and hence it may attempt to travel at the fastest speed possible. Equation 9(b) is when there is a vehicle ahead in which case we must study the relative motion of the two vehicles to compute the desirable speed. agg ($0 < agg \leq 1$) is the aggression factor. More aggressive driving is marked by higher acceleration and decelerations. A higher value of this factor means that the vehicle continues to drive at fast speeds, even after seeing the obstacle or vehicle ahead, and sharply decelerates (if needed) to avoid the obstacle or vehicle. Lower values imply a slower deceleration scenario.

Sometimes it may be possible that no potential collision is visible in the lateral direction, but the vehicle is oriented at some angle θ' such that it is close to an obstacle, vehicle, or boundary end. Hence at the same point f_i calculations are repeated with distances measured along the current heading angle of the vehicle or θ', which gives another preferential driving speed indicator $v_{\theta' i}$. The resultant preferred driving speed is given by (10).

$$v_{desired} = min\{v_{fi}, v_{\theta' i}\},\ i \text{ lies on } C_1C_2 \tag{10}$$

This speed may not be obtainable due to acceleration limits ($-accmax \leq acceleration \leq accmax.agg$), in which case the maximum change allowed is made.

IV. Results

The algorithm was developed and tested using a self-made simulation tool in MATLAB. For computational reasons we measured the various potentials only at the corners of the vehicle, instead of measuring them at every point across the vehicle boundary and then taking the maximum. Unless a small vehicle or obstacle happens to lie strictly in between the vehicle corners, which would be the case with very small obstacles or vehicles, this approach holds good. We generated a number of scenarios to test the working of the algorithm with respect to its parameters.

A. *Experimental Scenarios*

A variety of scenarios were created to assess the behavior of the vehicle. We first tested the obstacle avoidance capability of the vehicle. A single vehicle was created on the road which needed to overcome two obstacles one after the other. The path followed by the vehicle is shown in Fig. 2(a). The vehicle steered left to place itself so as to comfortably pass the first obstacle. Soon the second obstacle was detected, and on being close enough, steering took place on the opposite side.

The next scenario was created to test the ability of the vehicle to overtake another vehicle. To make the scenario difficult, a static obstacle was added just after potential overtaking completion. The green vehicle was capable of high speeds. It emerged later in the scenario, overtook the slower vehicle (red) at a point *A*, and proceeded to pass thee obstacle as shown in Fig. 2(b), while the red vehicle slowly moved on towards the obstacle. The red vehicle showed cooperation and drifted lefts to allow the overtaking procedure as denoted by point *B* in Fig. 2(b).

In the third scenario the red vehicle is first made to enter, which travels straight. Then green vehicle is then made to enter which simply follows the red vehicle, exhibiting vehicle following behaviors. Then the blue vehicle entered, which was capable of high speeds. It succeeded to overtake first the green vehicle (at point *A*) and then the red vehicle (at point *B*). This scenario is shown in Fig. 2(c).

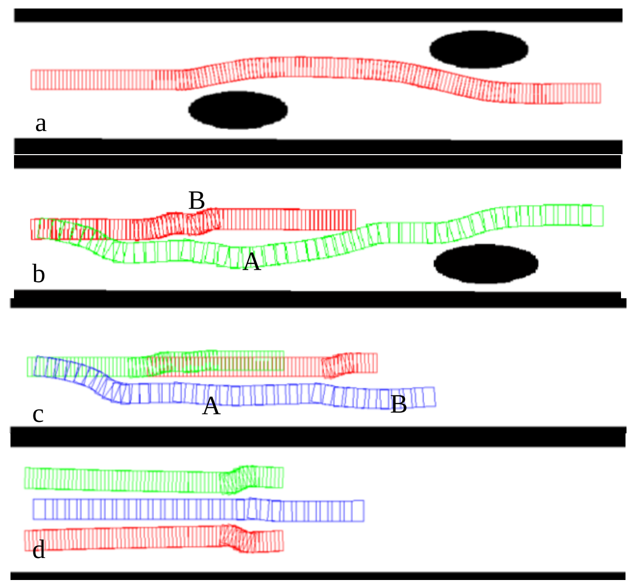

Fig. 2: Simulation results of the algorithm

In the last scenario two vehicles (red and green) entered the map simultaneously, separated laterally by some distance. The vehicles continued to move parallel to each other, with the same speed. Lateral potentials from each other and road boundaries made them drift towards each other, in order to make lateral separations equal. Later the blue vehicle entered the scenario and proceeded to firstly push the two vehicles and then it succeeded in intercepting them. Finally the blue vehicle overtook the two vehicles. The scenario is shown in Fig. 2(d).

B. *Algorithmic Parameters*

Equation (6) shows a number of parameters which govern the contributions of the various kinds of potential. One of the important parameters of the algorithm is $sen_{X'}$ which covers the sensitivity along the X' axis. This parameter was tested for the case of a single obstacle, in which the performance largely depends upon this parameter. The effect of different values on the path length is shown in Fig. 3(a). The paths corresponding to various values are shown in Fig. 3(b). It is clear that low values lead to late steering, while high values cause immediate steering and early positioning to avoid an obstacle.

The other parameter of interest is *coop*, which governs the magnitude by which a vehicle cooperates with other vehicles. A simple scenario was created with a slow moving vehicle ahead in road. A fast vehicle entering the scenario could simply overtake the slower vehicle. The magnitude by which the vehicle being overtaken cooperates with the faster vehicle is the magnitude by which it drifts on the road. This leads to less of a need for the overtaking vehicle to steer. The path length of the overtaking vehicle and the vehicle being overtaken for different values of the parameter *coop* are shown in Fig. 4(a). The path corresponding to some of the values is shown in Fig. 4(b). It can be seen that high values of this parameter are desirable for the overtaking vehicle and less desirable for the vehicle being overtaken.

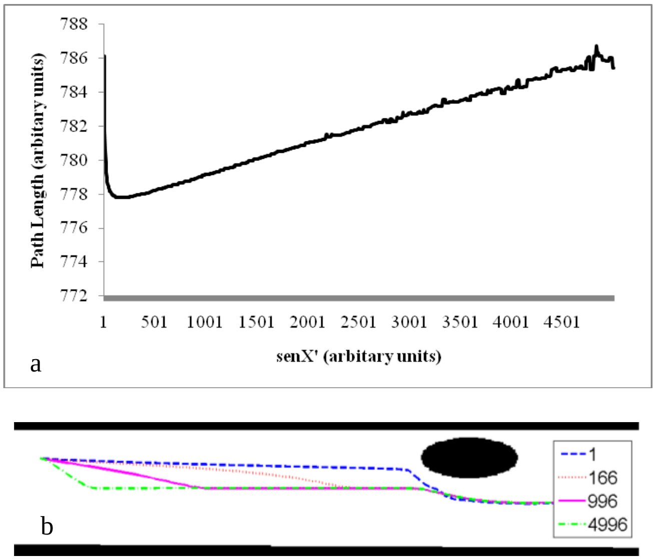

Fig. 3: Effect of changing parameter $sen_{X'}$

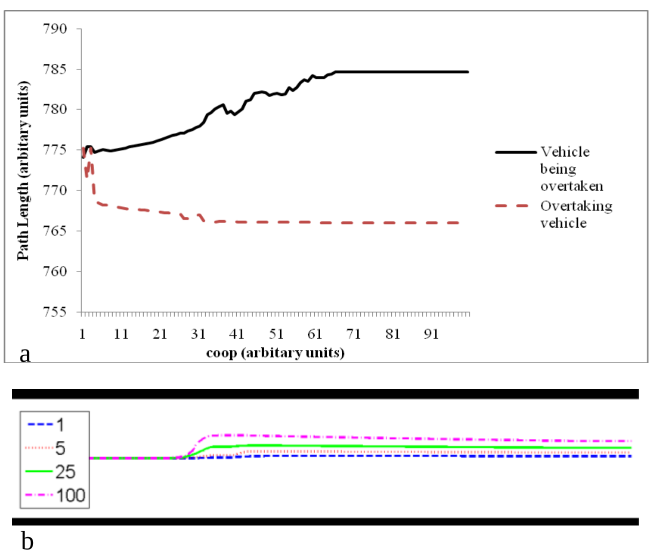

Fig. 4: Effect of changing parameter coop

The other important factor is $sen_{Y'}$, which governs the sensitivity of the vehicle in the Y' axis. We take a simple scenario with a single vehicle generated on the side of the road which would prefer to drift towards the center of the road due to unequal lateral potential by the road boundaries. The behavior of the vehicle for different values of this parameter is shown in Fig. 5. On further increasing the value of $sen_{Y'}$ the vehicle became highly sensitive and showed oscillations within the road.

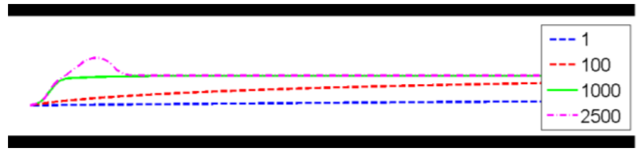

Fig. 5: Effect of changing parameter $sen_{Y'}$

V. Conclusions

The absence of speed lanes makes it difficult to plan autonomous vehicles and this can lead to chaotic traffic movement. The non-adherence to speed lanes in many countries provides benefits in terms of additional bandwidth for vehicles differing greatly in both their speed and width. Having no communication between vehicles further complicates the process.

In this paper we used lateral potentials as a solution to the problem. Vehicles, obstacles and road boundaries on all sides of the vehicle contribute to the potential and ultimately the steering of the vehicle. Each source of lateral potential is carefully chosen so as to lead to an overall vehicle behavior which is commonly found in chaotic traffic. The simulated results showed that a vehicle was able to avoid obstacles, navigate amidst other vehicles, and overtake other vehicles.

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