Motion Planning of Autonomous Vehicles in a Non-Autonomous Vehicle Environment without Speed Lanes

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Citation: R. Kala, K. Warwick (2013) Motion Planning of Autonomous Vehicles in a Non-Autonomous Vehicle Environment without Speed Lanes. Engineering Applications of Artificial Intelligence, 26(5-6): 1588–1601.

Final Version Available At: http://www.sciencedirect.com/science/article/pii/S0952197613000201

Abstract

Planning is one of the key problems for autonomous vehicles operating in road scenarios. Present planning algorithms operate with the assumption that traffic is organized in predefined speed lanes, which makes it impossible to allow autonomous vehicles in countries with unorganized traffic. Unorganized traffic is though capable of higher traffic bandwidths when constituting vehicles vary in their speed capabilities and sizes. Diverse vehicles in an unorganized exhibit unique driving behaviours which are analyzed in this paper by a simulation study. The aim of the work reported here is to create a planning algorithm for mixed traffic consisting of both autonomous and non-autonomous vehicles without any inter-vehicle communication. The awareness (e.g. vision) of every vehicle is restricted to nearby vehicles only and a straight infinite road is assumed for decision making regarding navigation in the presence of multiple vehicles. Exhibited behaviours include obstacle avoidance, overtaking, giving way for vehicles to overtake from behind, vehicle following, adjusting the lateral lane position and so on. A conflict of plans is a major issue which will almost certainly arise in the absence of inter-vehicle communication. Hence each vehicle needs to continuously track other vehicles and rectify plans whenever a collision seems likely. Further it is observed here that driver aggression plays a vital role in overall traffic dynamics, hence this has also been factored in accordingly. This work is hence a step forward towards achieving autonomous vehicles in unorganized traffic, while similar effort would be required for planning problems such as intersections, mergers, diversions and other modules like localization.

Keywords: Autonomous vehicles, traffic simulation, reactive planning, behavioural planning, decentralized planning, intelligent vehicles.

Classification codes: C78 Other computer applications, C33 Control applications, D2 Applications

Glossary

Scenario

$Boundary_1$: curve representing the left boundary
$Boundary_2$: curve representing the right boundary
XY: Cartesian coordinate system (X axis and Y axis)
X'Y': Road coordinate system (X' axis and Y' axis)
P(x,y): Any point in Cartesian coordinate system
P(x',y') or $P(p_x', p_y')$: Any point in road coordinate system
$Boundary_2(x')$: Point in Cartesian coordinate system located at a distance of x' along $Boundary_2$
$Boundary_1(x')$: Point in Cartesian coordinate system located in $Boundary_1$ across Y' axis at $Boundary_2(x')$
rl: Road length
||.||: Euclidian norm
abs(): Absolute value function

For i^{th} vehicle

$R_i(x_i, y_i)$ or $R(x_i', y_i')$: Position
θ_i: Orientation

len_i: Length
wid_i: Width
v_i: Speed
$vMax_i$: Maximum speed
$-accMax_i$: Maximum retardation
$accMax_i$: Maximum acceleration
$\Delta x'_{ij}$: Uncertainty in measurement of position of vehicle R_j by R_i
x'_{ij}: Possible positions of R_j as perceived by R_i
$\Delta v'_{ij}$: Uncertainty in measurement of speed of vehicle R_j by R_i
v_{ij}: Possible speeds of R_j as perceived by R_i
$\Delta(v_i)$: Visibility range for obstacles/other vehicles
$separMin_i$: Aggression factor, minimum separation which must always be planned to be maintained
$separMax_i$: Maximum separation beyond which separation maximization has no safety incentive
ζ_i^{free}: Free configuration space
l_i: Distance available to the left of R_i measured along the Y' axis
r_i: Distance available to the right of R_i measured along the Y' axis

Algorithm
OA: Point of obstacle avoidance
CEN(x',0.5): Point at centre of the road at a distance of x' in X' axis
curve(a,b): Spline curve joining a (with current orientation) to b (oriented along X' axis)
τ: Planned trajectory for vehicle motion
$\tau(t)$ or $(x^{t'}, y^{t'})$: Position of vehicle planned at time t
lane_change(τ): Lane change behaviour for trajectory τ
straighten(τ): Function to straighten a trajectory τ
side: 'left' or 'right'. Strategy to overtake a vehicle or avoid an obstacle

1. Introduction

As a result of the DARPA Urban Challenge (Buehler et al., 2007; Seetharaman et al., 2006), Google Cars (Marko, 2010), and similar projects (e.g. Pradalier et al., 2005, Sotelo et al., 2004), there has been an increased effort towards making vehicles autonomous and addressing the pertinent open issues. Such vehicles can not only add a lot of personal comfort for passengers but also considerably increase the safety of the overall traffic system (Bishop, 2000). However, realising vehicle autonomy opens up issues relating to obstacle detection, localization, control, planning, coordination, etc. (Montemerlo et al., 2008). Many approaches assume an inter-vehicle communication mechanism to solve the issues (Tsugawa, 2002; Reichardt et al., 2002), which however is not a practical assumption to make.

In most papers in the literature the presence of speed lanes is assumed and hence a vehicle planner is given the task of deciding when to change lane, the speed of travel, and a trajectory for lane changing. Broadly speaking, these models regard a decision to change between speed lanes as a high level decision, and speed control as a low level decision. High level decisions are then taken based on the positions of other vehicles and the feasibility and utility of a change (Schubert et al., 2010; Furda and Vlacic, 2011). Such decisions may be used to imitate overtaking behaviours (Naranjo et al., 2008; Hegeman et al., 2009) which play a major role in enabling vehicles to drive at high speeds. Low level decisions meanwhile are based more on safety measures.

Conversely, we advocate here that an optimal overall travel plan may not be possible with the presence of speed lanes. Diversity is an important factor that plays a role in making the employment of speed lanes non-optimal. As an example, diversity in vehicle widths makes it possible for multiple vehicles to lie within the same speed lane or to lie between speed lanes (pp 35, Sewall et al., 2011), both of which increase the bandwidth. Otherwise even small vehicles would have occupied a single speed lane thereby restricting the bandwidth to the number of speed lanes. Further, large variations in speed make it mandatory for higher speed vehicles to overtake lower speed vehicles as early as possible, as well as for lower speed vehicles to be overtaken by higher speed vehicles. In uniform traffic a vehicle has no problem in following another vehicle, however while driving in a car one would not like to follow a bicycle moving on the road. Our study is based on such a traffic scenario which may be easily termed as an unorganized traffic.

To motivate the notion we take a look at traffic on Indian roads. Vehicle widths may vary from two-wheeled motor bikes, to three-wheelers (including auto rickshaws), to 4-wheeled cars and 8-wheeled trucks, while the

speed varies from manually ridden bi-cycles to fast cars. As a result it is natural for drivers not to follow marked speed lanes, overtaking has become a fairly common sight, expectation from a smaller (slow moving) vehicle is to respect a larger (faster moving) vehicle and to allow it space to overtake, and vehicles may temporarily use the 'wrong side' of a dual carriageway to carry out an overtake (from the point of view of problems of traffic prediction, characteristics are also explained in Vanajakshi et al., 2009). While many readers may argue that these are not good traffic practices, we strongly emphasize here that in fact such an approach potentially leads to higher bandwidths and better traffic efficiency.

Driver aggression also has a role to play in the dynamics of an organized traffic which is limited to accelerating or decelerating one's own vehicle, risk prone cut-ins and speed lane changes, use of the horn, etc. (McGarva et al., 2000; Lajunen et al., 1999; Shinar and Compton, 2004). In the absence of speed lanes aggression plays an even greater role and governs critical decisions such as whether or not to overtake a vehicle in front.

Unorganized traffic has an apparent higher risk of accidents, and this needs to be a factor for consideration in design of a planning algorithm. We hence studied traffic accident causes for organized traffic (Clarke et al., 2005; Chalmers and Thomas, 2005; Wang and Xu, 2010) and unorganized traffic (Mohan et al., 1985; Mohan, 2002). For unorganized traffic, two wheelers are particularly prone to accidents (Clarke et al., 2004; Jain et al., 2009). Analysis reveals that most, if not all, of the head-on collisions on non-crossing scenarios are caused by factors absent in autonomous vehicles which have active intelligent sensing and actuation systems. However this does place emphasis on keeping a safe distance from the vehicle in front so as to avoid accidents even if the vehicle in front stops suddenly. Accidents may however be possible when one vehicle crashes into the side of another vehicle, producing slight damage to both vehicles. Excessive steering is one reason for such a crash which, in a non-speed lane system, may be produced due to an attempt to incorrectly fit into the available space or wrongly trying to give another vehicle space in the road.

It is clear that unorganized traffic is advantageous in a number of countries. The challenge is how to get autonomous vehicles into such traffic, knowing that the underlying algorithms are prone to work only with speed lanes. Whilst engineering the complete vehicle would require localization, vision, control, planning, etc. modules to be workable in unorganized traffic, only the problem of planning is considered in this paper. Further planning is restricted to a straight road scenario, while no effort is given to planning in cases of intersection, diversions, intersections, complete road blockages, etc.

In this paper the attempt is to study the possible behaviours in an unorganized traffic and further to model them in an algorithm. The objective behind the algorithm is to be able to move an autonomous vehicle flawlessly within unorganized traffic. The algorithm must not make the vehicle either wait for a long time to move ahead (starve) or to cause accidents. The algorithm must also allow the vehicle to move by means of the optimal route and speed, at the same time allowing faster vehicles to overtake, and the vehicle to overtake slower vehicles whenever possible. The algorithm is modelled as a collection of behaviours, it assesses the current situation and selects the most appropriate behaviour based on which the vehicle is moved at any instant.

The novelty of the work is (i) We study vehicle behaviours in unorganized traffic and identify behaviours. (ii) Each behaviour is modelled in an algorithm used for the motion of autonomous vehicles in unorganized traffic. (iii) Driver aggression is studied and modelled as an algorithmic parameter in such traffic. (iv) Simulation studies are carried out on a number of synthetic scenarios, based on which traffic dynamics of unorganized traffic are observed.

The closest related work is that of Paruchuri et al. (2002) who simulated vehicle behaviour on straight roads and crossings without traffic lights. They discussed vehicular behaviours where one vehicle could follow another vehicle, overtake another vehicle, or multiple vehicles could result in a traffic jam. The limitations include the assumption of inter vehicle communication, no cooperation between vehicles, the algorithm seemed to be lane-prone and the overtaking decision module did not generalize to a high number of vehicles with unorganized patterns. Meanwhile Leonard et al. (2008) chose Rapidly-exploring Random Trees (RRT) for planning a vehicle, although the algorithm was non-cooperative. Kala and Warwick (2011) used the RRT-Connect algorithm for planning multiple vehicles. However the algorithm assumed that inter-vehicle communication, was non-cooperative, and could not be scaled to a large number of vehicles. Furda and Vlacic (2011) used Automata by which they could show the behaviours of maintenance of position on the road, maintenance of a safe distance from other vehicles, collision avoidance, etc. The selection of behaviour was through Multi-Criterion Decision making. The assumption of speed lanes however make the behaviour set small and simple.

A behaviour based approach is common in the domain of multi robot path planning where the problem is to move the robots from their sources to goals. Most approaches (Aguirre and Gonzalez, 2000; Beom and Cho, 1995) use sensors that sense the environment around, assess the situation, and use fuzzy based methods to move the robot. While mobile robotics often considers open spaces as the map, traffic scenarios have predefined roads which are long and narrow (in comparison) - this makes a big difference. In traffic scenarios no particular point is regarded as the source and goal, and safety is an important concern. Hence these methods are different from road following behaviour in traffic scenarios, and they have different behaviours and dynamics.

In the work of Alvarez-Sanchez et al. (2010) every robot step was taken to maximize the distance from obstacles. In our paper we have used a similar idea whereby a vehicle attempts to maximize its separation from other vehicles. The maximization effort is however under a threshold and only lateral separations are considered. We further extend this basic notion to develop complex behaviours and cooperative measures, which was absent in their work. Jolly et al. (2010) modelled the various behaviours observed in robotic soccer. The rule set was built with discrete field regions, which were later learnt using a fuzzy neural network. Whilst driving exhibits more complex behaviours, driving behaviours are clearly discrete in nature.

Modelling of the planning scenario as well as the planning algorithm is discussed in section 2. Individual behaviours modelled in the algorithm are described in section 3. Simulation of the algorithm results in a number of interesting observations with the use of multiple vehicles. Such multiple behaviours are studied via simulations in section 4. Some concluding remarks are then given in section 5.

2. Problem and Solution Modelling

2.1 Problem Modelling

The basic problem considered here is to safely move a vehicle in a scenario amidst multiple robotic or manually driven vehicles. Hence consider a road segment which is characterized by its two boundaries say $Boundary_1$ and $Boundary_2$, lying on either side of the road. A vehicle R_i is assumed to be rectangular of length len_i and width wid_i. The speed of the vehicle is changed by acceleration (or retardation) over the interval [$-accMax_i$, $accMax_i$]. The maximum speed of any vehicle is fixed to a value of $vMax_i$

Although the general planner we have developed here works for all types of roads, having any kind of curves, this behaviour study is restricted in the first instance to a straight road. As a result, while initiating any behaviour a vehicle assumes the road ahead to be straight and infinitely long. The reason for this is that completion of a maneuver might involve cooperation of other vehicles about which the vehicle initiating the behaviour may never be sure. Since it will take a finite time for a manoeuvre to complete in cooperation with other vehicles, the vehicle needs to assume that the road is long enough for this. Further, the road is assumed to be straight in order to overcome decision making on dynamics based on the curve. Vehicles travelling on the same curve would differ in their time of travel depending upon their lateral position on the curve. Study of behaviours such as diversions, merging, and crossing scenarios are not within the scope of this current work.

In this approach we deal with two coordinate systems: the Cartesian coordinate system (XY) and a road coordinate system (X'Y'), which has the X' axis as $Boundary_2$ and the Y' axis is perpendicular to the X' axis. For any point P(x,y), the Y' coordinate (y') is the ratio of the distance of the point P(x, y) from $Boundary_2$ and the width of the road measured in the Cartesian coordinate system (equation (1)). This factor is also known as the speed lane occupancy of the vehicle. Notations are shown in Figure 1. The road coordinate axis system enables easier lane keeping in a curved road and anticipating other vehicles' positions on roads with irregular widths.

$$P(x,y) = P(x',y') = P\left(x', \frac{\left\| P(x,y) - Boundary_2(x') \right\|}{\left\| Boundary_1(x') - Boundary_2(x') \right\|} \right) \tag{1}$$

Each vehicle R_i keeps a track of all the vehicles in its neighbourhood. Tracking however is prone to be erroneous and such errors can be especially high if the other vehicle is at a distance. As per the uncertainty model used, the position of vehicle R_j as measured by vehicle R_i in the X' axis is given by equation (2) and the speed of vehicle R_j as measured by vehicle R_i is given by equation (3).

$$\Delta x'_{ij} = puMag\left(e^{puSpreadabs(x'_i - x'_j)} - 1 \right) \tag{2}$$

$$x'_{ij} = [x'_j - \Delta x'_{ij}, x'_j + \Delta x'_{ij}]$$

$$\Delta v'_{ij} = vuMag\left(e^{vuSpreadabs(x'_i - x'_j)} - 1\right) \tag{3}$$

$$v_{ij} = [v_j - \Delta v'_{ij}, v_j + \Delta v'_{ij}]$$

Here puSpread is the positional uncertainty spread and puMag is the positional uncertainty magnification factor. Similar factors for speed are vuSpread and vuMag.

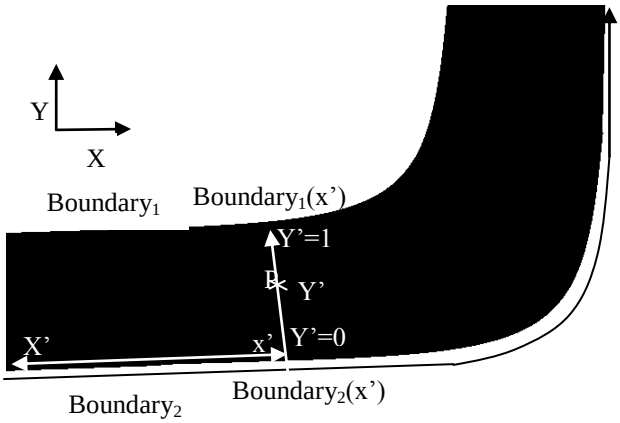


Figure 1: Road and Cartesian Coordinate Axis system

The visibility of any vehicle is restricted to a distance of $\Delta(v_i)$ along the length of the road. In the present implementation we keep this factor linearly proportional to the current speed of the vehicle v_i (or $\Delta(v_i) = k.v_i$). A vehicle would not be able to see any obstacle or another vehicle beyond this factor. In natural driving this factor is infinite for straight roads and depends upon the degree of curvature for non-straight roads. However the factor is restricted to the limits of the computational load of the obstacle discovery algorithm. We further assume that all roads are one way, in other words the roads are clearly divided for inbound and outbound traffic.

2.2 Aggression Factor

Aggression is a commonly used term to assess a driver's performance. Though there is no clearly defined metric to express how aggressive a driver is, aggression is qualitatively assessed with close cut-ins, too many lane changes, close overtakes, sharp speed changes and manoeuvres etc. The cons of aggressive driving include passenger discomfort, risk of accidents, and discomfort for other drivers who have to be more alert and reactive. The pros of aggressive driving include usually a shorter travel time for the aggressive driver. This advantage of the aggressive driver may trigger a corresponding disadvantage for a non-aggressive driver, who may have to take cautious steps to ensure safety causing their travel time to be longer. Traffic is an adversarial system, where the different vehicles may not be able to simultaneously exhibit their optimal plans or in other words the different vehicles compete to be near-optimal. Aggressive drivers are usually more towards a win situation (with associated cons of aggressive driving) while the less aggressive drivers are more towards the lose situation. As an example consider a scenario where a road narrows so as to allow only one of two competing vehicles to go forth. In such a scenario a more aggressive driver would normally make their way through. Of course if both drivers are very aggressive, a collision would occur.

One of the important tasks in the algorithm is to model aggression as an algorithmic factor. In all displays of aggressive driving we see that the driver keeps very little separation from other vehicles or obstacles. The distance from the vehicle directly ahead is always enough to ensure no collision if the vehicle suddenly stops, and it barely contributes to aggression. The separation which constitutes aggression is the separation that a vehicle plans to keep from the other vehicles and obstacles at its side. In a lane-based system this parameter is pre-set which is the width of a lane minus a vehicle's width. In a non-lane-based system, this is a parameter.

The algorithm must always plan to keep a separation of more than $separMin_i$ at side, where $separMin_i$ is called the aggression factor. A larger separation would be preferred for increased safety, even if it be at the expense of

the path length or travel time. However after a separation of $separMax_i$ the path is regarded safe enough and any attempt to increase separation further would only be done if it brings an incentive in terms of the path length or travel time. We study this here by an example. Consider a vehicle is driving behind another vehicle and wants to overtake it. If the vehicle behind cannot construct an overtaking trajectory, such that the separation of the projected motion is always greater than $separMin_i$ from sides, the overtake would not happen. However if the road is wide enough, the overtake would happen with the two vehicles separated by a lateral separation of approximately $separMax_i$. If the overtaking vehicle attempts to steer any further, it would make its path worse while adding no safety. If the road is not wide enough, the lateral separation would be kept to as large as possible.

2.3 Algorithm Modelling

The entire algorithm is modelled as a collection of behaviours that we need the vehicle to display. The different behaviours are inter-related to each other. The overall behaviour to be displayed depends upon the surrounding vehicles. In this manner the approach is similar to the use of a rule based system for navigation of the vehicle. In a system with speed lane the rules can only produce a single output indicating the speed lane to travel along. However in this paper speed lanes are not in play, hence many more behaviours must necessarily be exhibited. While some of the behaviours may sound similar to a speed-lane based system, the ability of a vehicle to showcase similar behaviours in the absence of speed lanes adds to the differences between our work and that of others.

Each behaviour has a set of conditions which must be true for the vehicle to display that particular behaviour. Even whilst one behaviour is being displayed the conditions for other behaviours need to be checked. Each behaviour has its own priority which is used to decide which behaviour needs to be executed when a number are possible. Our notion of behaviour is simple. Consider driving on the road. In some situation a driver may decide to overtake a vehicle ahead. In another situation it may be necessary to overcome an obstacle. These actions constitute different behaviours. At any time instant the decision taken results in the behaviour which is best for a vehicle in cooperation with the other vehicles. Here we do not showcase two such behaviours simultaneously, say overtaking a vehicle and avoiding an obstacle, an actual behaviour would be one of them only.

With the stated modelling scenario and assumptions, the set of behaviours are constructed with the following hypotheses:

i. No interesting behaviour may be initiated if it appears to have a separation less than $separMin_i$ in future. On the other hand if either currently or in the future it appears that the separations may drop below $separMin_i$, preventive measures need to be initiated.
ii. The aim of the algorithm is to attempt to increase the separation as much as possible (upto $separMax_i$). Hence obstacles and vehicles should be avoided by the maximum possible separations.
iii. In case a separation of more than $separMax_i$ is available, the vehicle must attempt to minimize its lateral movements (amount of steering). Hence obstacle avoidance and vehicle overtaking would happen from the sides which are closer to the vehicle. This also suggests that in general a vehicle would always attempt to travel parallel to the road.
iv. For plans involving the same lateral moments, a plan which steers to the new lateral position in a single attempt is preferable to a plan which does the same in multiple steering events.
v. A vehicle must attempt any possible safe overtake as well as cooperate to make a safe overtake possible.
vi. Every steering attempt must be as smooth as possible. However smoothness may be put under a threshold if it improves any other performance metric. Hence vehicles may be made to reasonably quickly align to overtake, to allow overtaking or to increase separations.
vii. In the long term a vehicle may attempt to maximize separations more than $separMax_i$ which means slowly drifting to the centre of the road on an empty road.
viii. At every instance the attempt is to have the maximum possible speed considered safe as per the situation.
ix. If a vehicle (by lane change) comes in front of another vehicle, it must give enough time for the other vehicle to adjust its speeds to avoid any possible accident.

3. Behaviours

3.1 Obstacle Avoidance

One of the most basic behaviours in the system is the ability of the vehicle to avoid an obstacle. Consider that the current position of the vehicle R_i is (x_i', y_i', θ_i). For trajectory generation, we first compute the best avoidance point considering all the obstacles that lie within $\Delta(v_i)$.

Consider the Y' axis at a distance of p_x' along the X' axis. Segments of this line would lie in regions without obstacles and segments within obstacles. A vehicle may either avoid an obstacle on its left side or right side. The vehicle simply selects the widest segment for its navigation. Let the widest segment be defined by the set of points [a, b] $\subseteq$ [$Boundary_2(p_x')$, $Boundary_1(p_x')$]. Within this segment, the vehicle must attempt to move such that it has to change its relative position in the Y' axis by a minimal amount and the obstacle is avoided by as large a separation (under a threshold of $separMax_i$) as possible in the Y' axis. By these objectives the point of avoidance P(p_x', p_y') at a distance of p_x' is given by equation (4).

$$p_y^{'} = \begin{cases} y' & a \le y'.rl - wid_i/2 - separMax_i \le y'.rl + wid_i/2 + separMax_i \le b \quad (i) \\ (a+b)/2rl & b - a \le wid_i + 2separMax_i \quad (ii) \\ (a + separMax_i + wid_i/2)/rl & a > y'.rl - wid_i/2 - separMax_i \quad (iii) \\ (b - separMax_i - wid_i/2)/rl & b < y'.rl + wid_i/2 + separMax_i \quad (iv) \end{cases} \tag{4}$$

Here wid_i is the width of the vehicle and rl is the width of the road given by

$$rl = \| Boundary_1(x') - Boundary_2(x') \|$$

Equation 4(i) covers the scenario where no obstacle lies in the vehicle's normal path at a distance of $separMax_i$ from both sides. In some cases the segment might however be too small for a vehicle to move without the desirable maximum separation, in which case it simply tries to maximize the separation by driving right at the middle as stated in equation 4(ii). In case the segment is wide enough to allow a separation of more than $separMax_i$ on both sides and the vehicle by its normal course does not enjoy this separation, planning is done so that the least amount of steering or change in relative position is required. If the point of avoidance lies towards the left side of the infeasible segment the scenario is given in equation 4(iii) and if it lies on the right side of the feasible segment the scenario is given in equation 4(iv). The various cases are depicted in Figure 2. The additional line shown is the planned trajectory which is not actually traced because an obstacle is encountered in the line of motion.

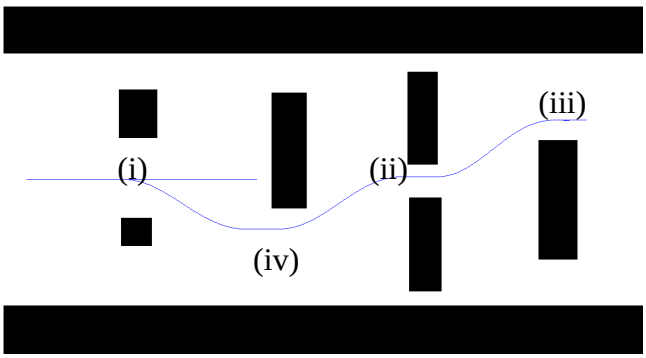


Figure 2: Computation of avoidance point in obstacle avoidance strategy

Out of all the competing points the best obstacle avoidance point OA is chosen which is the rightmost point for a right turn obstacle avoidance (or leftmost for a left turn obstacle avoidance) along the considered road length (x_i' to l). Selection of the rightmost (or leftmost) point assures that no steering is required for obstacles in the road length beyond OA while a vehicle should be able to drive to OA with minimum separation. In other words the intent is to steer as much as possible and then drive straight. The trajectory of a vehicle τ from its current position R_i to OA is constructed using spline curves. Every point $\tau(t)$ on the curve must be in the free

configuration space ζ_i^{free} of the vehicle (which accounts for a minimal separation of $separMin_i$) for the trajectory to be accepted. The procedure is given by Algorithm 1.

Algorithm 1:
ObstacleAvoidance(R_i, map)
Calculate $P(p_x', p_y')$ $\forall$ $p_x' \in [x_i', x_i'+\Delta]$ as per equation (4)
$l \leftarrow x_i'+\Delta$
while true
    $OA \leftarrow P_1(p_{1x}', p_{1y}')$: $abs(p_{1y}'-y') > abs(p_{2y}'-y')$ $\forall$ $p_{1x}', p_{2x}' \in [x_i', l]$, $P_1 \neq P_2$, $P_1, P_2 \in P$
    $\tau \leftarrow curve(R_i, OA)$
    if $\tau(t) \in \zeta_i^{free}$ $\forall$ t, return τ
    else $l \leftarrow OA(x') - 1$

3.2 Centring

Even though it might be comfortable for the vehicle to travel with its current relative position y' in the Y' axis, it may alternatively plan to slowly drift towards the road centre. Since the drift is slow, it doesn't require a large steering effort on the vehicle's part. Advantages of driving in the centre include easier adaptation to bends/curves and variations in road widths. In order for this behaviour to be initiated, there must be no slower vehicles or obstacles ahead of the vehicle along the road length $\Delta(v_i)$. The vehicle must itself be travelling at a reasonable speed, which ensures that this behaviour is not invoked at low speeds, when it is not relevant. The vehicle chooses the point of aim at the centre of the road width or $CEN(x_i'+\Delta(v_i), 0.5)$. Spline curves are used for trajectory generation.

3.3 Lane Change

A lane change in itself is not a behaviour but rather a precondition which must be true for a number of behaviours including obstacle avoidance and centring. In other words it may be seen as a super behaviour which incorporates a number of behaviours in itself. The enabling condition of this super behaviour leads to the enabling conditions of the sub-behaviours. A possible lane change is queried when any vehicle needs to change its relative position y' by a reasonable amount.

Every vehicle, as a safety precaution, needs to ensure that it keeps a sufficient safety distance from the vehicle in front, considering its own acceleration limits. This means that even if the vehicle in front was to stop suddenly, no collision would occur. However this does not account for the scenario wherein another vehicle might suddenly cut in from the side thereby forcing the vehicle to re-compute its speed and adjust accordingly. If the vehicle is travelling at a good speed and a vehicle suddenly comes in front from the side, it may not be able to decelerate fast enough to avoid a collision. Even if only a part of the vehicle cutting in lies ahead of the vehicle in question, a collision is possible.

Hence any change in lane is allowed only if the vehicle can complete the entire trajectory of lane change without needing any vehicles in the rear to slow down. Consider that the vehicle R_i needs to change its speed lane as per the trajectory τ. In doing so it changes its relative position (or lane) from y_i' to $y^{t'}$. The time required by the vehicle to do so is given by equation (5).

$$t = \| \tau \| / v_i \tag{5}$$

Here $\| \tau \|$ indicates the total length of the curve τ and v_i is the current speed of the vehicle. It is assumed that no other vehicle lies within the speed lane occupancy of y_i' to $y^{t'}$ of R_i.

Vehicle R_i assumes that all other vehicles continue to move at the same speed with the same relative positions as per the uncertainty model till time t. Hence the predicted path of a vehicle from the current time to time t may be given by equation (6) which is an extension of equations (2) and (3).

$$x_{ij}^{'}(t) = \left[x_j^{'} - \Delta x_{ij}^{'} + \left(v_j - \Delta v_{ij}^{'}\right)t, x_j^{'} + \Delta x_{ij}^{'} + \left(v_j + \Delta v_{ij}^{'}\right)t\right] \tag{6}$$

It is assumed here that while vehicle R_i travels on its trajectory τ, other vehicles plan as per equation (6). If this results in a potential collision, vehicle R_i will need to take an alternative behaviour. It is evident that only vehicles having any part of their vehicle within the lane occupancy of y_i' to $y^{t'}$ of R_i need to be checked. Further

if vehicle R_i has a smaller operating speed than any of the other vehicles it might have to lower its speed in order to avoid a collision. However it would have sufficient time to do this.

3.4 Overtaking

Since vehicles potentially have large differences in speed, it may not be 'fair' for a vehicle R_i with high speed capability to follow a slow vehicle. Hence if it is possible it tries to overtake the slower vehicle in front. The vehicle in front is firstly assessed. Any part of the vehicle should lie within the relative position of [y_i'-(wid_i/2+$separMin_i$)/rl, y_i'+(wid_i/2+ $separMin_i$)/rl] ahead of vehicle R_i. The term denotes the speed lane occupancy of R_i with minimum thresholds at both ends. Here rl is the current road width. While multiple vehicles are potential candidates for the vehicle R_i to overtake, it is essential to select one such vehicle only. Hence we select the vehicle which is closest to the vehicle R_i (in terms of its position in the Y' axis) unless it lies far apart in terms of distance in the X' axis. In simple terms this means that a vehicle a long way ahead is not as significant as a vehicle a reasonable distance ahead, whilst a vehicle directly in front is preferred over a closer vehicle which is to the side. This means it is not only the vehicle directly in front which needs to be overtaken, but also any vehicle that is in front of the vehicle at a distance of $separMin_i$ from both sides.

Consider that the vehicle in front is R_j. Let l_j be the distance available to the left of R_j measured along the Y' axis (beyond which some other vehicle or road boundary lies) and r_j be the distance available to the right of R_j. Note that if no vehicle is in front, the vehicle does not exhibit this behaviour. Two scenarios arise - direct overtaking and assistive overtaking.

3.4.1 Direct Overtaking: Direct overtaking can take place by a vehicle R_i when there is sufficient distance on either the left or right hand side of another vehicle R_j so that R_i may simply steer and slip in. In order to slip in R_i requires a minimum distance of wid_i+2.$separMin_i$ considering the minimum separation that it must maintain from both ends. The first decision to be made however is the side that overtaking will take place - that is whether R_i overtakes R_j on the left or right. Mostly overtaking on the right is what happens in countries which drive on the left and vice versa. However in a situation with wide lanes and no speed lanes overtaking on the right may not always be possible. The decision only needs to be made when sufficient distance from both sides is available else the side having sufficient distance can only host the overtake. A simple overtaking rule is defined here, which states that in the case when R_i is more towards the left side of vehicle R_j, overtaking takes place on the left and vice versa. If R_i is exactly behind R_j overtaking takes place on the right.

The other task to be carried out is to decide on the position or the point of overtake P(p_x',p_y') to which the vehicle must travel. The trajectory τ is calculated until a point P(p_x',p_y') on the road traversing which the overtaking would have happened, or the vehicles would be aligned, so that travelling straight would complete the overtake. We break the decision separately into the computation of p_x' and p_y'. The value p_y' denotes the relative position of the vehicle R_i on either side of the vehicle R_j as decided. The same concepts are used as noted in equation (4). The value of p_y' is given by equation (7).

$$p_y^{'} = \begin{cases} (y_j^{'}.rl + wid_j/2 + separMax_i + wid_i/2)/rl & l_j \geq wid_i + 2separMax_i \wedge side = left \quad (i) \\ (y_j^{'}.rl + wid_j/2 - l_j/2)/rl & l_j < wid_i + 2separMax_i \wedge side = left \quad (ii) \\ (y_j^{'}.rl - wid_j/2 - separMax_i - wid_i/2)/rl & r_j \geq wid_i + 2separMax_i \wedge side = right \quad (iii) \\ (y_j^{'}.rl - wid_j/2 - r_j/2)/rl & r_j < wid_i + 2separMax_i \wedge side = right \quad (iv) \end{cases} \quad (7)$$

Equation 7(i) holds when a separation of more than $separMax_i$ is available on the left side of vehicle R_j and left is the overtaking side. If the same distance is not available and overtaking needs to take place on the left side, vehicle R_i simply attempts to maintain as large a distance as possible to other vehicles. Similarly equation 7(iii) is when separation $separMax_i$ is available and equation 7(iv) when this separation is not available. Notations are shown in Figure 3 for a random scenario for which one condition of equation (7) may be active.

The value of p_x' is the distance across the X' axis within which the vehicle should align itself and attain the value of p_y' along the Y' axis. Normally the overtake wouldn't have been completed when the vehicle reaches the point P(p_x',p_y') but this behaviour has more to do with arranging the relative positions of all vehicles and facilitating conditions such that R_i may travel straight ahead at high speed. Hence we allow enough distance along the length of the road for R_i to steer as per needs. A higher deviation in relative position from the current position along the Y' axis means a larger distance along the X' axis for alignment. Further, high speed may

require large distances along X' axis for the same magnitude of steering. The value of p_x' may then be given by equation (8).

$$p_x' = c_1 + c_2.v_i + c_3.\ abs(y' - p_y').rl \quad (8)$$

Here c_1, c_2, c_3 are constants. c_1 is the minimal distance across the X' axis needed to produce a smooth curve as per spline curves. Usually this is set to be twice the length of the vehicle. c_2 and c_3 meanwhile denote contributions of the factors of speed and steering requirements.

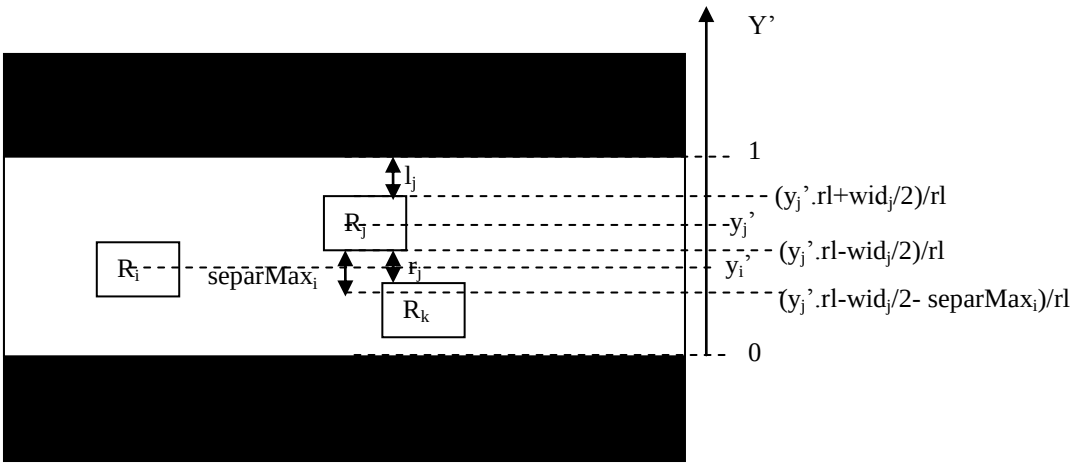


Figure 3: Notations used for a random scenario (all measurements in Y' axis)

3.4.2 Assistive Overtaking: The second scenario possible is assistive overtaking wherein an overtake may still be possible but it requires the assistance of other vehicles. For the overtake to be possible the other vehicles must rearrange themselves by steering and changing their relative positions on the road, thereby allowing R_i adequate space to overtake. In order to check whether such a scenario is possible we iteratively traverse both the left (and right) hand side of the vehicle in front R_j, at every iteration moving to the vehicle on its left (and right). Subtracting a basic constant amount equivalent to the presumed minimal width for every vehicle, we get a rough idea of the maximum separation available. If vehicle R_i itself comes into any iteration, it is not considered. If this space is more than wid_i+2.$separMin_i$, the vehicle may decide to overtake. Whether an overtake actually happens or not is uncertain as vehicles may have larger minimum thresholds and not offer enough room for R_i to pass through. It is even possible that vehicles may have smaller thresholds, and hence a feasible overtake is not attempted.

Further it is possible that multiple vehicles might simultaneously want to overtake with no one vehicle having enough separation of its own to overtake. We do not model here cooperative behaviour in which all vehicles decide which one overtakes, but rather we consider an aggressive behaviour in which they all attempt to overtake. It is quite possible therefore that because of the aggression no vehicle is able to actually overtake at a specific time. The decision whether to initiate overtaking or not is also critical. If the vehicle decides not to overtake, it displays a simple behaviour such as following vehicles in front of it. However if it tries to overtake, it constantly pushes other vehicles to give it more space so as to make its desired overtake possible.

If an overtaking procedure is initiated, R_i must choose its relative position in terms of alignment in the Y' axis such that some time later it might overtake easily if possible. Once more it might overtake a vehicle directly in front (R_j) either from the left or right side, decided purely on the basis of whether it is more towards the left or right of the vehicle. We compute the point of overtake P(p_x',p_y') for the case of assistive overtaking. The value of p_x' is kept the same as it was in equation (8). Considering the minimum distance to be available, the value of p_y' is given by equation (9).

$$p_y' = \begin{cases} (y_j'.rl + wid_j/2 + separMin_i + wid_i/2)/rl & side = left \\ (y_j'.rl - wid_j/2 - separMin_i - wid_i/2)/rl & side = right \end{cases} \quad (9)$$

Trajectory generation τ is done by the use of a spline curve. Once the trajectory τ is constructed the task for the vehicle is to travel along it. While the trajectory is planned and fixed, the only thing to be monitored and changed is the speed so as to avoid any collisions. The attempt is to make the vehicle travel with the highest speed possible. Hence at any time while executing the trajectory let the vehicle in front be at a distance of d units in front measured from the front of the vehicle R_i to the back of the vehicle in front. The minimum separation of $separMin_i$ needs to be maintained and further it needs to be assumed that the vehicle in front may stop suddenly. The maximum deceleration is taken to be $-accMax_i$. Hence if v_i is the current speed of the vehicle, the changed speed at the next instant v_i' is given by equation (10)

$vm = \sqrt{(2.accMax_i.\max(d - separMin_i, 0))}$

$$v_i^{'} = \begin{cases} \min(v_i + accMax_i, v_i^{\max}) & v_i + accMax_i \le v_m \\ \max(v_i - accMax_i, vm) & v_i > v_m \\ vm & otherwise \end{cases} \quad (10)$$

Here vm is the maximum speed as per the distance available.

Whichever is the scenario of overtaking, the other vehicles in the vicinity need to be made aware of the vehicle R_i attempting to overtake. This information helps them in their planning. In the case of an assistive overtake the other vehicles need to move and create space as we will see later. In the case of direct overtaking vehicles need to be prevented from accidently moving in the opposite direction and reducing the available separation. For manual vehicles this may be in the form of a horn or similar gesture commonly used in traffic while overtaking. Autonomous vehicles may additionally have some alternative inter-vehicle communication. This is the only bounded communication expected/allowed in the simulation. The overtaking gesture is updated at every decision making cycle. Whenever a vehicle finds a situation not suitable for overtaking in any manner, the gesture is stopped. This would be, for example, when the vehicle has already completed an overtake and vehicles ahead are now to the side, or when the vehicles in front have left enough room for the vehicle R_i to comfortably drive straight ahead, or the overtake is later found infeasible. The infeasibility may be due to a reduction in road spacing or perhaps a new vehicle entering parallel to the stream of vehicles ahead.

Another prerequisite is that vehicles which are being overtaken must be travelling straight or on the opposite side (left or right) from which the overtaking is being initiated. This resolves any conflict of interest wherein a sufficient separation is seen and the vehicle initiates overtaking only to find that the vehicle in front continuously moves to the opposite side thereby reducing the available separation. If either of the scenarios does not hold or vehicles are not seen to be travelling straight or on the opposite side or a change of lane request is denied, no overtaking is possible.

3.5 Be overtaken

A related behaviour along with overtaking is to be overtaken. In assistive overtaking a vehicle expects the vehicle in front to give it more separation. Hence whenever possible the vehicle in front must cooperate and allow the vehicle behind to overtake. For this behaviour a vehicle R_i scans all the vehicles requiring to overtake behind it within a relative position of $[y_i'-(wid_i/2+separMax_i)/rl, y_i'+(wid_i/2+separMax_i)/rl]$. Note the use of $separMax_i$ in place of $separMin_i$. This is done for 2 reasons. Firstly the vehicle behind say R_j may have a higher value of $separMin_i$, which means that even if the vehicle R_i sees the vehicle R_j having enough distance to go ahead, it might actually not proceed. To enable an overtake the vehicle R_i must further give more space for vehicle R_j to overtake. Secondly giving more space than the minimal amount is always considered better as per the modelling problem. Out of all the available vehicles the vehicle R_i selects the vehicle R_j which is closest to it in terms of separation in the X' axis. R_i can easily 'guess' the side which R_j wants to overtake by the same rules.

Now R_i must give the necessary separation to R_j for overtaking, or at least the maximum that it can give. In case it does not have adequate separation, further space may be given by a vehicle to the side of R_i or various vehicles may steer themselves and move in due course of time (as we shall see in later behaviours) giving more separation to R_i to allow space for R_j. Let l_i be the separation available with R_i on its left and r_i be the separation available on its right. Assuming that the separation thresholds (minimum and maximum) of the two vehicles are the same, the separation needed by R_j may be anywhere in the range $[wid_j+2.separMin_i, wid_j+2.separMax_i]$, which needs to be created within the separation available with R_i which is (l_i+r_i), also considering the minimum separation requirements of R_i.

Say the vehicle R_j needs to overtake R_i on the right, which means R_i must drift leftward to facilitate the overtake. Since R_i drifts leftwards, the space r_i is already available for R_j to overtake, making the requirements within the range [a, b] = [wid_j+2.$separMin_i$ - r_i, wid_j+2.$separMax_i$ - r_i]. The preferred relative position of R_i to account for the overtake of R_j is given by equation (11).

$$p_y^{'} = \begin{cases} NIL & b \le 0 & (i) \\ (y'.rl + l_i - separMin_i)/rl & a > l_i - separMin_i & (ii) \\ (y'.rl - r_i + 2.fs + wid_j)/rl & b > l_i - separMin_i & (iii) \\ (y'.rl - b)/rl & otherwise & (iv) \end{cases} \tag{11}$$

Here fs is the free space after both R_i and R_j fit in side by side. It is given by

fs = l_i+r_i-wid_j

Equation 11(i) is the case when no assistance from R_i is required and there is sufficient space (with $separMax_i$ distance additional at both sides) available for R_j on the right side of R_i. Equation 11(ii) deals with the case when R_i is unable to give sufficient space, keeping the minimum separation reserved for itself and hence it can give the maximum separation possible to R_j. Equation 11(iii) is the case when the separation available is not large enough so that R_j can enjoy a separation of $separMax_i$ from both their ends. Hence the available separation (fs) is equally divided between R_i and R_j. The case in equation 11(iv) is when R_i can offer a separation of b to R_j which is the maximum that it needs.

A similar equation is used when R_j overtakes R_i on the left side. The process of selecting an equivalent p_x' and using this as the space creation point is similar to the method discussed earlier. Similarly the trajectory is generated and the vehicle is moved. Again caution needs to be taken that distances are only measured if vehicles are travelling straight or drifting to the side needed by R_i. Lane change behaviour is checked for every vehicle other than R_j which is overtaking.

3.6 Maintain Separation Steer

This behaviour attempts to maximize a vehicle's separation from both sides, under a threshold of $separMax_i$, in cases where steering may increase the separation. Say the vehicle R_i has a separation of l_i units on the left and r_i units on the right. Separations are regularized by selecting a regularization point P(p_x', p_y') whose p_y' value is given by equation (12). Other principles are the same as those considered with the lane change super behaviour.

$$p_y^{'} = \begin{cases} NIL & l_i + r_i < 2separMin_i \vee l_i > separMax_i \wedge r_i > separMax_i & (i) \\ (y'.rl + l_i - separMax_i)/rl & l_i + r_i \ge 2separMax_i \wedge l_i < separMax_i & (ii) \\ (y'.rl - r_i + separMax_i)/rl & l_i + r_i \ge 2separMax_i \wedge r_i < separMax_i & (iii) \\ (y'.rl + (l_i - r_i)/2)/rl & l_i + r_i < 2separMax_i & (iv) \end{cases} \tag{12}$$

Equation 12(i) is operative when the behaviour is not applicable; that is when the vehicle enjoys a separation of more than $separMax_i$ on both sides or when it cannot enjoy a separation of $separMin_i$ on both sides. Equations 12(ii) and 12(iii) satisfy the condition when a vehicle enjoys a separation of more than $separMax_i$ on one side and less separation on the other side, while the total separation is such that the vehicle can enjoy its threshold value $separMax_i$ on both sides. Equation 12(iv) attempts to centre the vehicle by maintaining equal separation on both sides.

3.7 Slow Down

If a vehicle, by any steering mechanism, cannot enjoy a distance of more than $separMin_i$ on both sides (that is l_i+r_i<2.$separMin_i$), there is a potential threat of collisions due to uncertainties, possible control errors, etc. This situation marks the state of the vehicle where it is not 'comfortable' driving due to the very low separation available. Whenever this situation is encountered, vehicle R_i decreases its speed by its a maximum amount making the new speed as v' = max(v_i - $accMax_i$, 0). Since the dynamics of the road, in terms of the relative

positions of vehicles, is constantly changing it is possible later that the separation needed will be available and the vehicle can accelerate again. Further, as there is a possibility that a vehicle going ahead may create more space so that other vehicles can adjust and create space for the vehicle R_i. In case the separation is continuously low, R_i could instead join the stream of vehicles behind it, or follow the stream of vehicles in front. Note that a slowing down of vehicle R_i would mean its separation being eventually distributed across the stream of other vehicles.

3.8 Discover Conflicting Interests

Planning without formal communication makes it very likely that two vehicles showcase behaviours which, when executed simultaneously, lead to collision. Consider the scenario in Figure 4, the two vehicles compute and aim to occupy the same space in front, and hence their trajectories, when executed, would lead to a collision. Such a situation rarely happens in daily life that two vehicles simultaneously make opposing plans. If one of the vehicles had been a little late in decision making, the other vehicle would have displayed its intention to turn. This intention was a consideration in all the behaviours involving separations.

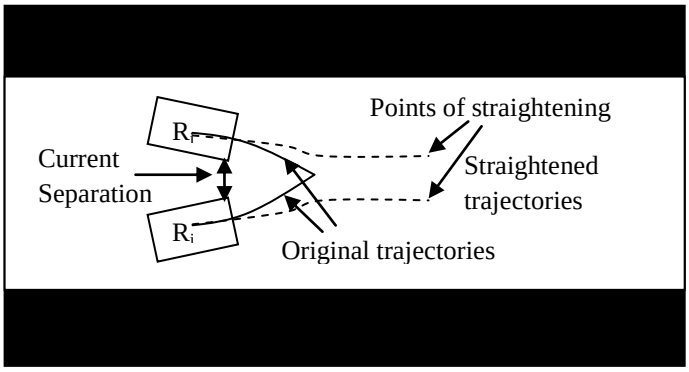


Figure 4: Discover conflicting Interests Behaviour

The smallest separation between any two vehicles is always monitored while one vehicle traverses its planned trajectory τ. If this distance happens to be such that after unit move of both vehicles the separation would be less than the minimum threshold, vehicle trajectories are straightened. The point of straightening is computed whose value in the Y' axis is found by measuring the least subsequent motion in the Y' axis before the vehicle can straighten itself which is the projection of the vehicle's length in the current Y' axis.

3.9 Travel Straight

The last behaviour to be considered is in fact the simplest behaviour as this deals with travelling straight on the road. There are no prerequisites for this behaviour, it can be invoked at any time and is usually invoked when there are no other behaviours to be exhibited. A small curve is generated for a unit move of the vehicle from the current position, keeping the relative position (y') of the vehicle constant. The vehicle attempts to move by the highest speed possible given by equation (10).

The various behaviours discussed here are summarized in Table 1. Note that many of the behaviours are specific in terms of their initiation and hence a change of their priority would not affect the algorithm. Priorities are useful only when multiple behaviours may simultaneously meet their preconditions.

The algorithm as a pseudo code is given by algorithm 2. Here the various behaviours have been listed one after the other. Lane change behaviour is marked by lane_change(τ) which returns true if a change is possible. This characteristic also appears in most other behaviours. If a behaviour results in the generation of a trajectory τ, the same is returned in the plan, which becomes the input for the next time instance. A non-null value of τ signifies a behaviour being followed and the same behaviour should be followed unless it has been completed. In all cases where τ is null, a new behaviour is searched for. Behaviours which complete in unit move (such as moving straight) return a null value of τ, signifying a behaviour was selected and was completed in the same step.

Table 1: Summary of vehicle behaviours

S. No.	Behaviour	Pre-Condition	Description	In-behaviour specifications	Priority
1.	Obstacle Avoidance	Obstacle discovery, lane change true	Strategy to avoid obstacle	Check for collisions with vehicle in front, obstacle avoidance	1
2.	Centring	No vehicle, no obstacle, vehicle moving by large speeds	Put vehicle in road's centre	NIL	2
3.	Lane Change	Called by other behaviours	Whether possible to steer	NIL	NA
4.	Overtake	Slower vehicle ahead, sufficient separation available assuming cooperation of all vehicles ahead, lane change true	Strategy to initiate overtake, ask other vehicles to move, and eventually align so that travelling straight completes overtake	Discover Conflicting Interests, Check for collisions with vehicle in front	3
5.	Be overtaken	Vehicle at back shows need of overtaking, separation available to offer, lane change true	Align so that vehicle at back needing to overtake has more overtaking separation	Discover Conflicting Interests, Check for collisions with vehicle in front	4
6.	Maintain Separation Steer	Maximum separation possible not available at both ends while steering in some manner can increase current lowest separation, lane change true	Steer to maintain as high separation as possible (not more than threshold) from both ends	Discover Conflicting Interests, Check for collisions with vehicle in front	5
7.	Slow Down	No adjustment of steering capable of generating minimal separation at both ends	Reduce speed	NIL	6
8.	Discover Conflicting Interests	A neighbouring vehicle steering towards vehicle being planned found too close while the vehicle being planned was steering towards it, vehicle following a non-straight trajectory	Straighten trajectory being followed	Check for collisions with vehicle in front	NA
9.	Travel Straight	NIL	Take a unit step forward as per road's current orientation	Check for collisions with vehicle in front	7

Algorithm 2:
Plan(Vehicle R_i, Map, Previous plan τ)

If new obstacle found
    Compute τ by Algorithm 1
    If lane_change(τ), return τ
    Else, $v_i \leftarrow \max(v_i - accMax_i, 0)$, $R_i \leftarrow$ move a unit step, return null
If no slower vehicle and obstacle ahead in vicinity $\wedge$ v_i close to v_i^{max} $\wedge$ τ = null
    CEN $\leftarrow (x_i' + \Delta(v_i), 0.5)$
    $\tau \leftarrow$ curve(R_i,CEN)
    If $\tau(t) \in \zeta_i^{free}$ $\forall$ t, return τ
If $\tau \neq$ null
    $v_i \leftarrow$ Safe speed as per equation (10)

If Conflicting Interests $\wedge$ τ is non-straight
$\tau \leftarrow$ straighten(τ), return τ
Else
$R_i \leftarrow$ Move a unit step by τ
If τ is over, return null, else return τ
If slow vehicle in front $\wedge$ sufficient separation exists for overtaking assuming cooperation
$R_j \leftarrow$Vehicle to overtake, side $\leftarrow$ side of overtaking
Compute P from equations (7), (8) and (9)
$\tau \leftarrow$ curve(R_i,P)
If $\tau(t) \in \zeta_i^{free}$ $\forall$ t $\wedge$ lane_change(τ) $\wedge$ R_j not steering towards side, return τ
If vehicle overtaking at back $\wedge$ separation available to offer
$R_j \leftarrow$Vehicle to allow overtake, side $\leftarrow$ side of being overtaken
Compute P from equation (11)
$\tau \leftarrow$ curve(R_i,P)
If $\tau(t) \in \zeta_i^{free}$ $\forall$ t $\wedge$ lane_change(τ), return τ
If $l_i + r_i \geq 2.min\_separ_i$ $\wedge$ (l_i <max_separ$_i$ $\vee$ r_i <max_separ$_i$)
Compute P by equation (12)
$\tau \leftarrow$ curve(R_i,P)
If $\tau(t) \in \zeta_i^{free}$ $\forall$ t $\wedge$ lane_change(τ), return τ
If $l_i + r_i < 2.min\_separ_i$
$v_i \leftarrow$ max(v_i - accMax$_i$,0), $R_i \leftarrow$ move a unit step, return null
$v_i \leftarrow$ Safe speed as per equation (10)
$R_i \leftarrow$ Move a unit step parallel to the road
return null

4. Simulation Results

The simulation tool was developed in MATLAB. Base modules carried the task of boundary determination, vehicle tracking, graphical display, distance measurements. Different behaviours were coded as separate modules. Scenarios could be generated by specifying entry times, initial speeds, maximum speeds, maximum accelerations, and sizes of vehicles. The map was supplied as an image file. The algorithm was tested for various scenarios which are discussed in subsequent sections.

4. 1 General Traversal

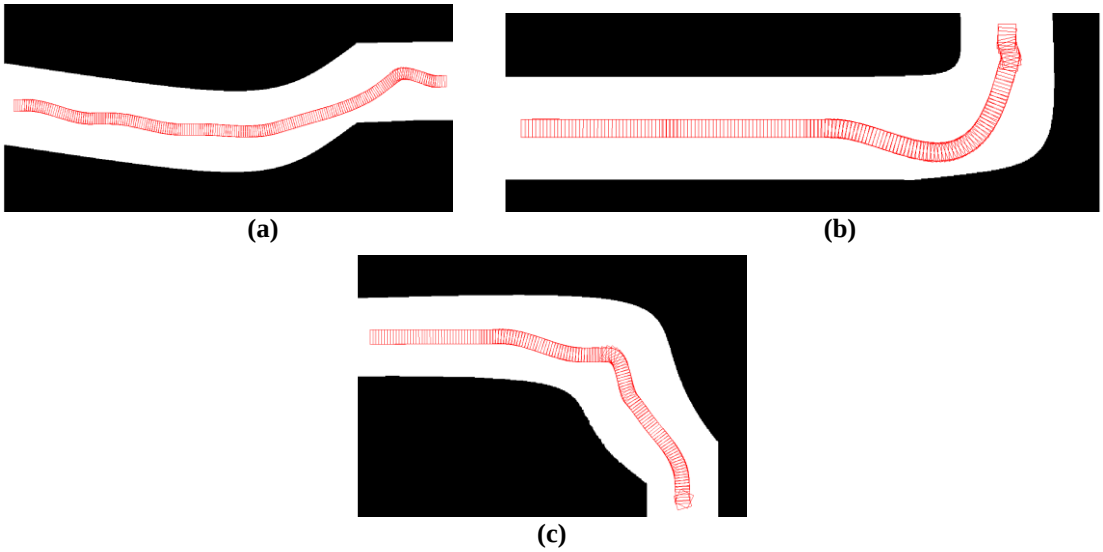
The first task associated with the testing of the algorithm was assessing the ability of a vehicle to travel along any general road. A number of maps were given to a single vehicle for its traversal. All these maps involved some challenges as to the method of taking a turn or the smoothness of the overall path. The results in three of the scenarios, each having a different road orientation, are presented in Figure 5. Since only one vehicle was travelling in the scenario, encouragement was given to centring.

Figure 5(a) shows a gradual turn where the vehicle was supposed to turn smoothly, before encountering a major straightening of the road. Clearly the straightening was sudden and non-smooth and we see that the vehicle was able to handle this by its planning, making the entire curve relatively smooth to follow with high speeds. Figure 5(b) depicts quite the opposite behaviour where the road is almost straight with a sudden curve right at the end. Such roads are a common phenomenon especially in crossing like situations. Figure 5(b) shows the path of the vehicle which was as smooth as possible in making the turn. Figure 5(c) meanwhile shows a road where the complete road segment may be broken down into further segments. The challenge was not so much to travel within segments but rather to steer between segments. While the vehicle makes a smooth steer in the first change, the manner of handling the second change is unique and no major steering is visible.

4.2 Obstacle Avoidance

The next major challenge associated with autonomy in vehicles is to be able to avoid any static obstacles, while still maintaining a smooth trajectory. The challenge becomes even more difficult when the vehicle is to avoid an obstacle in the presence of a curved road or another vehicle. We have studied here various possibilities via simulations, the results of which are presented in Figure 6. Figure 6(a) shows a road with a general curve and an obstacle. The vehicle was supposed to plan its smooth trajectory such that it is centred at the road end and the obstacle is avoided comfortably. The same analysis for a straight road and single vehicle is shown in Figure 6(b). The same scenario was again used for an additional vehicle and the trajectory of the two vehicles is shown

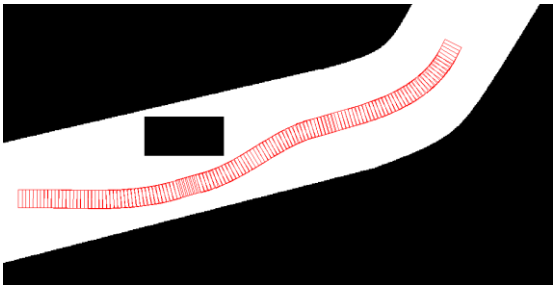
in Figure 6(c) and Figure 6(d). Note that the vehicle which originated lower in the scenario was generated later. This vehicle needed to drive straight, keeping its relative distance constant to avoid the obstacle. Hence here a straight driving behaviour was applicable rather than obstacle avoidance. The first vehicle, generated in the middle of the road, saw the obstacle ahead but was not allowed to change lanes to overtake it, as is shown in Figure 6(c). Hence instead it slowed down, waiting for the other vehicle to cross. It then passed the obstacle, as shown in Figure 6(d).



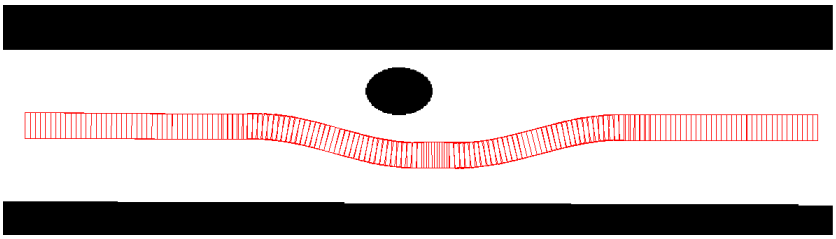
(a) **(b)**

(c)

Figure 5: General traversal of vehicle

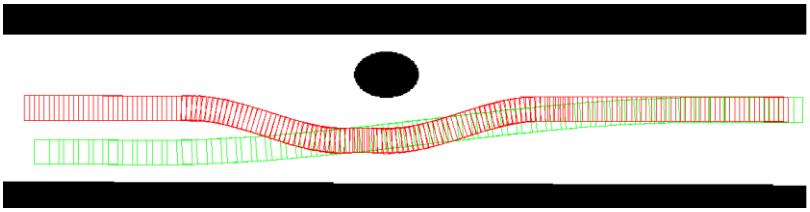


(a)



(b)

(c)



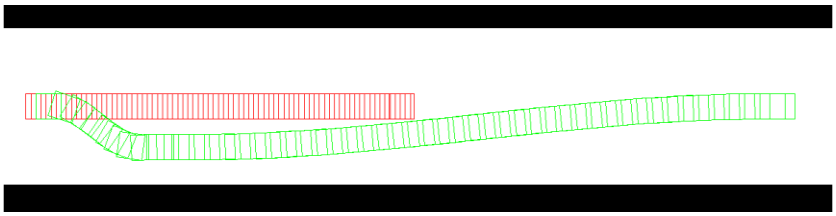
(d)

Figure 6: Obstacle Avoidance

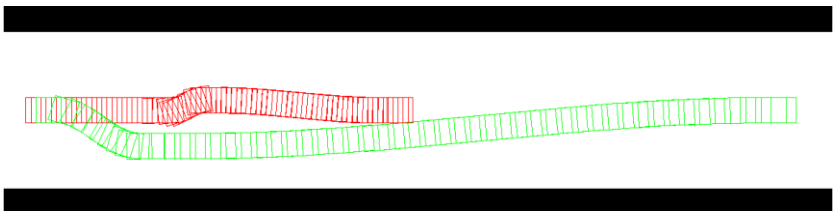
4.3 Overtaking

The next behaviour we explicitly studied was overtaking. We were interested primarily in vehicular behaviours which are best visible on a straight road but which are also common scenarios in travelling. The simplest scenario is when a vehicle comes from behind and attempts to overtake the vehicle in front with a lot of space to do so. The corresponding trajectory is shown in Figure 7(a). The overtaking vehicle keeps the maximum possible distance possible as per its set value of $separMax_i$ while overtaking. It quickly uses a trajectory of centring due to seeing no other vehicle in the vicinity, apart from the vehicle it has already overtaken. The same scenario was repeated with the overtaking vehicle being more aggressive than the vehicle being overtaken (in terms of its $separMax_i$ value). The trajectory in such a case is shown in Figure 7(b). Here the vehicle being overtaken feels it is possible to increase separation and be safer for the brief period of time the vehicle is overtaking. This is because of its less aggressive nature, more cautious driving, indicated by a larger value of $separMax_i$.

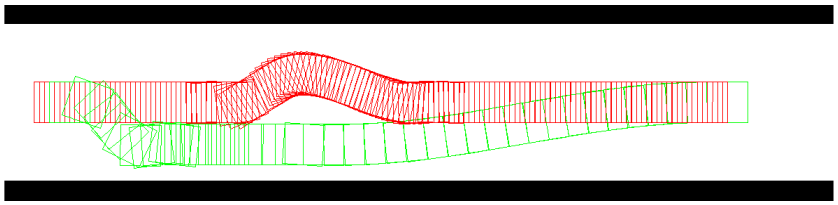
The last case dealt with was when the overtaking vehicle didn't have enough separation to overtake. The vehicle being overtaken was hence expected to move while the overtaking vehicle aligned itself. As soon as the vehicle attempting to overtake obtained enough separation, so overtaking was initiated. The corresponding trajectory is given in Figure 7(c). Figure 7(c) shows the importance of cooperation by the vehicle being overtaken to the overtaking vehicle. Overtaking is important to allow the two vehicles to drive at their maximum speeds as it might be 'painful' for a fast vehicle to follow a slow vehicle. Cooperation of the vehicle being overtaken first makes it possible to make overtaking possible and allows the vehicle to benefit from the same. Secondly it eliminates the need of the overtaking vehicle to steer by very large amounts which would make overtake a cumbersome job. Cooperation of the vehicle being overtaken by drifting leftward by some amount does not make its travel any worse; on the other hand it allows the overtaking vehicle to overtake easily and smoothly.



(a)



(b)



(c)

Figure 7: Overtaking Behaviour

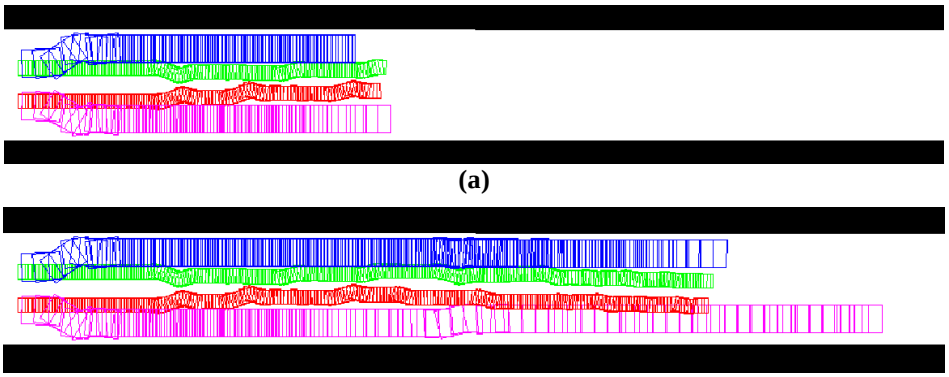
4.4 Complex Formulation

Based on the experimental results presented in the previous sub-sections, basic vehicular behaviours are well-understood. Hence many vehicles may be simulated over a road segment where each vehicle displays its individual behaviour at every time instant. We have identified one of the unique behaviours and wish to use this for our discussion. The complete scenario is shown in Figure 8 and is also presented in Video 1. In this simulation all vehicles have a different value of aggression factor, indicated by $separMin_i$.

The first two (smaller) vehicles were generated simultaneously and travelled parallel to each other on the road. After some time two more (larger) vehicles were generated simultaneously. The newly generated vehicles were capable of higher speeds and attempted to overtake the slower vehicles. The faster and larger vehicles travelled on either side of the road, forcing the two smaller vehicles to create some separation for them. Each of the vehicles computed the total available space and believed that overtaking was possible, which might actually not be the scenario as two vehicles are simultaneously trying to overtake, each blocking the chances of the other vehicle. The smaller vehicles attempted to create some separation and moved to the opposite side, each unaware of the contradictory plans of the other vehicle. The vehicles soon displayed conflicting interests and straightened their trajectories.

The motion henceforth was purely driven by aggression factors. The top smaller vehicle was less aggressive (having a higher value of $separMin_i$) and attempted to maintain a larger separation (equal in magnitude to its $separMin_i$) from the bottom small vehicle. Hence it steered slightly to the top/left when the desired magnitude of separation was not available. However the bottom small vehicle was more aggressive with a smaller value of $separMin_i$. It saw the separation created as a possibility to create more separation for the bottom bigger vehicle. When the bottom vehicle moved towards the top so the top vehicle again saw that the separation available was less than the magnitude desirable. So it steered more towards the top. The bottom vehicle hence kept pushing the top vehicle, till the required separation was available for the bottom large vehicle to overtake, accounting for its additional minimum separation value.

Once the required separation was available the bottom large vehicle overtook as shown in Figure 8(a). The centring behaviour was disabled and the vehicle travelled straight. Upon completion of overtaking there was ample separation available for the smaller bottom vehicle and it easily created a separation of $separMax_i$ for itself on both sides. However the small top vehicle was still struggling to create necessary separation for the larger vehicle behind it. It steered towards the bottom thereby reducing the separation of $separMax_i$ for the bottom small vehicle. The bottom small vehicle further steered towards the bottom to maximize the distance. In this manner reverse pushing occurred as the top vehicle pushed the bottom vehicle to create separation for the large vehicle at the rear. As soon as the required separation along with the minimum separation threshold was available for the larger vehicle, overtaking took place as is shown in Figure 8(b). Finally the two smaller vehicles travelled straight, parallel to each other.



(a)

(b)

Figure 8: Complex Formulation

We repeated the same experiment, but this time the value of the aggression factors remained constant. This scenario is naturally symmetric and hence no vehicle was capable of pushing another vehicle as was witnessed in Figure 8. The result was that the smaller vehicles kept looking for an opportunity to give the larger vehicles more separation, but they were unable to do so for the entire journey. The scenario is shown in Figure 9 and video 1.

Figure 9: Complex formulation with constant aggression factors

4.5 Aggression

In order to judge the working and effectiveness of the aggression factor we considered another scenario. A narrowing road was constructed with three vehicles travelling on it. It is evident that if all three vehicles travelled parallel to each other, none would be able to reach the end of the segment. However if aggression factors are different for the vehicles, it would be expected that more aggressive vehicles would reach the end, whilst less aggressive vehicles would fear danger at some time and would give way to other vehicles, allowing them to go ahead. To maximize the effect all vehicles were given generally high values of $separMin_i$. One case is discussed here and shown in Figure 10. The bottom vehicle was most aggressive followed by the central and top vehicle. Hence reasonably early in the journey both the central and top vehicles waited for the bottom vehicle to go ahead.

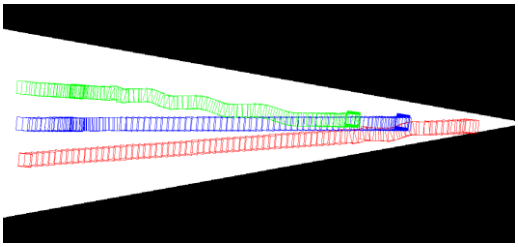


Figure 10: Experiments with the aggression factor

5. Conclusions

The objective of the research described here was to construct a planning algorithm for an autonomous vehicle amidst other non-autonomous vehicles and obstacles. The motivation was to solve the problem of planning along straight roads for autonomous vehicles driving in countries without speed lanes. Further effort was to advocate the absence of speed lanes for a diverse traffic system which autonomous vehicles may bring in the future. The absence of speed lanes brought about a rich set of behaviours which were identified and modelled.

The overtaking behaviour is particularly exciting, wherein it is difficult to decide which set of vehicles facilitate the overtaking of which other vehicles. At any time the vehicle could display any of the behaviours depending upon its state and the conditions that govern the execution of the behaviour. As in natural driving, aggression is another important factor which affects traffic dynamics. Experiments were performed for a variety of scenarios. Vehicles were tested for their ability to avoid obstacles, go around curved roads, overtake, help other vehicles overtake, and perform complex formulations.

Being primarily a reactive technique, some oscillations were found, especially in centring behaviour on curved roads, discovering conflicting interests behaviour, and prior to and post completion of overtaking. Such oscillations may be controlled using some deliberation and predicting the scenario of road and other vehicles in the future. The approach needs to be extended to other planning problems such as planning in intersections, mergers, etc. and planning in the case of special incidents such as the sudden breakdown of the vehicle in front, a road block, etc. Another limitation of the current work is that it has neither been validated on real traffic data sets nor is entirely evolved from real data sets. Currently we are however unaware of any such data sets for unorganized traffic. Unlike organized traffic, vehicles in unorganized traffic are difficult to track and dynamics are difficult to record. Recording and studying such real traffic would enable validation of the method as well as observing new interesting behaviours.

This study should particularly be helpful in overtaking decision making. Natural vehicles may be seen to give preference to certain vehicles which are more socially accepted or may use other indicators to make decisions

tussle-free between vehicles. Similarly real data sets would reveal more realistic modelling of aggression and how it affects the traffic landscape. Planning represents one of the many modules of autonomous vehicles, and all such modules need to be able to perform in the absence of speed lanes to enable vehicles to operate in real traffic. The major research issue is designing and validating the performance of other modules in the absence of lanes.

Acknowledgement

The authors wish to thank the Commonwealth Scholarship Commission in the United Kingdom and the British Council for their support of the first named author through the Commonwealth Scholarship and Fellowship Program - 2010 - UK award number INCS-2010-161.

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